

BANKING LIQUIDITY PARADOX: INSIGHTS FROM AN OPTIMAL RESERVE MODEL

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Abstract: The banking liquidity paradox, characterized by excessively liquid banks despite the insufficient supply of business loans, is a persistent concern in Sub-Saharan Africa (SSA). This paradox stems from both involuntary and voluntary factors influencing commercial banks' excess reserve holdings. Involuntary causes include substantial foreign currency inflows resulting from exports of commodities like oil, coffee, and cocoa. These export revenues significantly boost bank liquidity, a phenomenon observed in regions like CEMAC. Additionally, factors such as accumulating foreign exchange reserves to maintain currency stability in fixed parity monetary zones, remittances from migrants, official development aid, and debt relief initiatives like the Heavily Indebted Poor Countries Initiative (HIPC) contribute to increased bank liquidity. Furthermore, the repatriation of capital after currency devaluation and the establishment of regional stock exchanges amplify capital inflows. Voluntary factors also play a role, including restrictions on central banks financing national treasuries, as well as monetary policies associated with mandatory reserves. This article delves into the intricate web of involuntary and voluntary influences driving the banking liquidity paradox in SSA, shedding light on the complexities of the region's financial landscape.

Keywords: banking liquidity paradox, Sub-Saharan Africa (SSA), excess reserves, capital inflows, monetary policy, financial landscape.

1. Introduction

The question of the banking liquidity paradox in the banking system concerns the insufficient supply of business loans by banks, which are nevertheless excessively liquid. This shortfall in the global supply of loanable funds has been variously interpreted. In general, the causes of excess liquidity in the countries of Sub-Saharan Africa (SSA) are grouped into two categories: involuntary causes and voluntary causes of excess reserve holding by commercial banks (Agénor, Aizenmann and Hoffmaister, 2004). The involuntary detention of excess liquidity is explained by large inflows of foreign currency generated by the exports of certain commodities such as oil, coffee, cocoa, etc. The revenues from these exports inflate the liquidity of banks as studies show it for the CEMAC zone (Beguy, 2012; Doumbia, 2011). Other sources of increased bank liquidity are: the accumulation of foreign exchange reserves to defend the parity in fixed parity monetary zones via the internal and external stability of the value of the currency (Doumbia, 2009), the influx of funds from migrants, official development aid, the cancellation of the debt of certain countries following the Heavily Indebted Poor Countries Initiative (HIPC Initiative), the repatriation of capital after the devaluation. Other factors contribute to the influx

of capital: the establishment of regional stock exchanges, the prohibition of the financing of national treasuries by the central banks and the monetary policy resulting from the play of mandatory reserves. The involuntary detention of excess liquidity is explained partly by the underdeveloped nature of their financial market. For example, in a context of administered interest rates, banks are reluctant to grant loans that increase the risks of insolvency, instability and non-bankable projects (Eboué, 1990, 1998b). This results in credit rationing like the one studied by Modigliani and Jaffee (1969). The excess liquidity resulting from credit rationing can also be explained by information problems between lenders and borrowers (Stiglitz and Weiss, 1981). These information problems manifest themselves in the form of asymmetric information: the bank does not know the real quality of the projects (Stiglitz and Weiss, 1981) or in the form of symmetrical ignorance (Stiglitz and Emran, 2007): the borrowers themselves even ignore the quality of their projects.

This behavior of the banks is all the more marked as the lending rates (cost of credit) are very high. This increases the borrower's probability of default, the default rate and the reserve rate (Vo Thi, 2005, Prao, 2012). Banks are therefore becoming very cautious about loans granted. Other factors that increase credit rationing are the weak legal, judicial and regulatory framework (Sacerdoti, 2005), the lack of bankable projects, gaps in accounting standards, and the existence of a poorly developed justice system (Dolumbia, 2011). The reduction of these uncertainties involves the production of reliable accounting documents, the development of the client-agent relationship via proximity and trust, an update of accounting and auditing standards. Excess liquidity, linked to the holding of voluntary liquidity, is also justified by the high level of deposits by governments in some countries and by deficient loans (Saxegaard, 2006).

The voluntary holding of liquidity meets the desire of secondary banks facing growing uncertainty to avoid potential risks. This precautionary banking behavior resulting in excess liquidity has various consequences, particularly with regard to the effectiveness of monetary policy. Nissanke and Aryeetey (1998) show that bank excess liquidity weakens the transmission mechanism of monetary policy. More specifically, it becomes difficult, in the presence of excess liquidity, to regulate the money supply via the reserve requirement ratio and the monetary multiplier. Saxegaard (2006) tests this result for a sample of SSA countries. His study suggests that liquidity weakens the ability of monetary authorities to influence the conditions of demand in these countries. Agénor, Aizenmann and Hoffmaister (2004) obtain similar results. Ideas are proposed to absorb the excess bank liquidity by minimizing the uncertainties that cause the precautionary behavior of banks. Beguy (2012) proposes the establishment by the State of a guarantee fund allowing banks to recover part of their debts in the event of default. In doing so, this guarantee fund absorbs the banks' excess liquidity. For some, this entails a good restructuring of the judicial system in terms of efficiency to encourage banks to increase business loans (Pagano and Bianco, 2005). Another measure according to Beguy (2012) is the tax bonus. Indeed, for him the State can encourage the Banks to grant the credits by the implementation of a fiscal bonus, to those who will commit the most to the financing of the private sector.

Thus, works that address the issue of voluntary excess liquidity have explained it essentially by the risky environment faced by risk averse banks. These uncertainties are due to financial markets imperfections, information asymmetries, weak judicial and regulatory framework, etc. Our objective, in this paper, is to show that, whatever the degree of uncertainty (low uncertainty or high uncertainty), excess liquidity

may appear. This is the case when expected portfolio return of the risk neutral bank is sufficiently low. To address this issue, we take over a theoretical analysis, unlike most of the studies about excess liquidity that use empirical analysis. More precisely, we develop a model of optimal behavior of the bank. This model is an adaptation of Baumol's (1952) optimal cash model. The rest of the article is structured as follows: Section 2 models the optimal behavior of a risk-neutral bank. Then, in section 3, the hypothesis of a risk-averse bank makes it possible to highlight the main role of uncertainties in absorbing the excess banking liquidity. Section 4 concludes the paper.

2. The optimal behavior of the risk neutral bank

2.1. The assumptions

A risk-neutral bank realigns its portfolio once a period and is assumed to invest its total fund T in two types of assets (loans and risk-free securities). The bank decides how much of its portfolio to allocate to loans (x_{it}), risk-free securities (y_{it}) and liquidity ($1 - x_{it} - y_{it}$) in order to maximize its profit (Π_{it}). Loans, if they generate a higher expected return than that of risk-free securities, also induce more risk. Loans have two types of risk: market risk and default risk (credit risk). Loan yields (rc_{it}) and security assets (rs_{it}), the two types of assets that compose the bank portfolio, are:

$$\forall i, \forall t, \quad rs_{it} = r_f \quad (1)$$

$$\forall i, \forall t, \quad rc_{it} = r_f + \rho + \varepsilon_{it} \quad (2)$$

Where r_f : risk free interest rate; ρ : market risk premium ; $\varepsilon_{it} \sim N(0, \sigma_{\varepsilon_t}^2)$ is an idiosyncratic shock that affects the bank i at period t .

Noting x_{it} the proportion of the portfolio invested in loans by bank i in period t and $(1 - x_{it})$ the proportion of the portfolio invested in securities, the return of the bank portfolio is determined as follows: $\forall i, \forall t, \quad R_{it} = x_{it}rc_{it} + (1 - x_{it})rs_{it}$. This expression comes down to:

$$\forall i, \forall t, \quad R_{it} = r_f + x_{it}(\rho + \varepsilon_{it}) \quad (3)$$

The bank, supposedly risk neutral, optimizes its profit. We notice C_i : the volume of bank loans granted by the bank i at t ; A_{it} : the volume of security assets; i_c : the cost of the bank loan (interest rate); i_{MM} : the cost of refinancing on the money market; REF_{it} : the refinancing of bank i with the central bank in t . The profit of the bank is the difference between its total revenue and its total cost:

- Total revenue: revenues from bank loans ($i_c C_{it}$) and capital gains reported by the investment of security assets ($r_f A_{it}$);
- The total cost is the sum of the total refinancing cost ($i_{MM} REF_{it}$) and the total cost of holding the liquidity (CT_L).

The profit of the bank i at period t is written:

$$\Pi_{it} = i_c C_{it} + (r_f A_{it}) - i_{MM} REF_{it} - CT_L \quad (4)$$

To determine the total cost of holding liquidity, we draw on Baumol's optimal cash management model (1952), which we adapt to banking behavior. A bank must therefore hold at all times a level of liquidity such that it is not short of cash. Indeed, it must at all times satisfy the withdrawals of funds from its customers either at the counters or automated teller machines (ATMs). Insufficient liquidity can lead to insolvency of the bank. At the same time, however, this amount of liquidity should not be too large, because there is an option cost to holding it: the rate of return on the investments in which it could be

invested. It is assumed that the cash flows of the bank are certain, liquidity outflows are at a constant rate. The average liquidity held by the bank is $L/2$ (With L : bank liquidity at the beginning of the period). The bank's assets portfolio (loans, security assets) can be used to regenerate bank liquidity when needed. When liquidity becomes insufficient, the bank sells part of its portfolio at the beginning of the period to bring liquidity back to the desired level. In doing so, it regenerates its liquidity in the next period. The rate of decline in liquidity is constant over a period.

Each conversion of securities into cash corresponds to transaction costs (commission paid by the bank to its broker for the sale of securities, time spent by the bank on such transactions). Therefore, the lower the bank liquidity, the higher the number of conversions and the higher the transaction costs of the bank. Let F be the fixed costs that the bank incurs each time it converts securities into money. We have: $F = f_c C_{it} + f_A A_{it}$, where f_c are the fixed costs related to the conversion of risky securities into money and f_A are the fixed costs related to the conversion of safe securities into money. During a period, the total transaction costs of a bank related to the management of its liquidity are the product of the number of conversions that multiplies the fixed costs per conversion. With the additional assumption that the total fund the bank must have at period t is a multiple m of deposits it has in its reserves, $T_{it} = m(1 - r)D_{it}$. Thus, we have:

$$\text{Transaction cost} = \left(\frac{T_{it}}{L}\right) F = \left(\frac{m(1 - r)D_{it}}{L}\right) F \quad (5)$$

In addition, the holding of liquidity also includes an option cost. Cash does not pay any interest. The option cost related to the holding of cash therefore corresponds to the interest income sacrificed as a result of the conversion of securities into cash. If R_{it} is the rate of return on the bank portfolio from which the cash is generated, and since the average annual cash position of the bank is $(L/2)$, the option cost of holding the liquidity is as follows:

$$\text{Option cost} = \left(\frac{L}{2}\right) R_{it} = \left(\frac{L}{2}\right) [r_f + x_{it}(\rho + \varepsilon_{it})] \quad (6)$$

The total cost of holding the liquidity (CT_L) is the sum of the transaction cost and the option cost:

$$CT_L = \left(\frac{m(1 - r)D_{it}}{L}\right) F + \left(\frac{L}{2}\right) [r_f + x_{it}(\rho + \varepsilon_{it})] \quad (7)$$

2.2. The optimization program of the risk-neutral bank

The profit of the bank becomes:

$$\Pi_{it} = i_c C_{it} + r_f A_{it} - i_{MM} REF_{it} - \left(\frac{m(1 - r)D_{it}}{L}\right) F - \left(\frac{L}{2}\right) [r_f + x_{it}(\rho + \varepsilon_{it})] \quad (8)$$

We can refine this expression of the bank profit. To do this, we start from the fact that the refinancing of bank i with the central bank is the difference between, on the one hand, the sum of banknotes in circulation of the bank (B_{it}), the reserve requirements of bank i (RO_{it}) and, on the other hand, the sum of the value of the gold and currencies of the bank i (OD_{it}) and the net loans of the bank i to the treasury (T_{it}). So: $REF_{it} = B_{it} + RO_{it} - OD_{it} - T_{it}$. In addition, it is assumed that the deposits of bank i with the central bank consist only of reserve requirements RO_{it} whose rate is such that $RO_{it} = rD_{it}$. The net

contribution of the bank to the treasure at period t is: $T = FME_{it} - CCP_{it}$ with CCP : total postal check accounts, hence $REF_{it} = B_{it} + rD_{it} - OD_{it} - FME_{it} + CCP_{it}$. Finally, knowing that the money supply created by the bank i in t (M_{it}) is the sum of bank loans C_{it} , the monetary financing of the treasury FME_{it} and the value of gold and currencies OD_{it} ; we obtain $M_{it} = C_{it} + FME_{it} + OD_{it} \Rightarrow -OD_{it} - FME_{it} = C_{it} - M_{it}$. Because of this, $REF_{it} = B_{it} + rD_{it} + C_{it} - M_{it} + CCP_{it}$. It is assumed, moreover, that the bank bears interest on deposits ($i_D D$) with i_D the interest rate on deposits, and some variable costs whose growth rate g increases with the activity of banks (here measured by the distributed credit): $CV_{it} = gC_{it}^2$. Let us define the following ratios: $p = \frac{D_{it}}{M_{it}}$; $p' = \frac{B_{it}}{M_{it}}$ and $1 - p - p' = \frac{CCP_{it}}{M_{it}}$. We can write: $B_{it} + rD_{it} + C_{it} - M_{it} + CCP_{it} = p'M_{it} + rpM_{it} + C_{it} - M_{it} + (1 - p - p')M_{it} = (rp - p)M_{it} + C_{it}$. Remembering that $F = f_C C_{it} + f_A A_{it}$, $T_{it} = m(1 - r)D_{it}$, and $T_{it} = x_{it}C_{it} + y_{it}A_{it} + (1 - x_{it} - y_{it})L_{it}$, the bank profit is written:

$$\Pi_{it}(C_{it}, L_{it}, A_{it}) = i_C C_{it} + r_f A_{it} - i_{MM}[(rp - p)M_{it} + C_{it}] - i_D D_{it} - gC_{it}^2 - \left(\frac{x_{it}C_{it} + y_{it}A_{it} + (1 - x_{it} - y_{it})L_{it}}{L_{it}} \right) (f_C C_{it} + f_A A_{it}) - \left(\frac{L_{it}}{2} \right) [r_f + x_{it}(\rho + \varepsilon_{it})] \quad (9)$$

Finally, after some refittings, the expression of the bank profit is refined as follows:

$$\Pi_{it}(C_{it}, L_{it}, A_{it}) = [i_C - i_{MM}(1 + rp - p) - i_D p]C_{it} - (i_{MM}p(1 - r) + i_D)(FME_{it} + OD_{it}) + r_f A_{it} - gC_{it}^2 - \left(\frac{x_{it}C_{it} + y_{it}A_{it} + (1 - x_{it} - y_{it})L_{it}}{L_{it}} \right) (f_C C_{it} + f_A A_{it}) - \left(\frac{L_{it}}{2} \right) [r_f + x_{it}(\rho + \varepsilon_{it})] \quad (10)$$

The bank chooses the triplet $\{C_{it}, L_{it}, A_{it}\}$ so as to maximize the profit $\Pi_{it}(C_{it}, L_{it}, A_{it})$. The first order conditions give the following results:

$$C_{it} = \frac{i_C - i_{MM}(1 + rp - p) - i_D p - \frac{x_{it}f_A A_{it}}{L_{it}} - y_{it}A_{it}f_C - (1 - x_{it} - y_{it})L_{it}f_C}{2g + \frac{x_{it}f_C}{L_{it}} + x_{it}f_C} \quad (11)$$

$$L_{it} = \frac{2(f_C C_{it} + f_A A_{it})(x_{it}C_{it} + y_{it}A_{it})}{r_f + x_{it}(\rho + \varepsilon_{it})} \quad (12)$$

$$A_{it} = \frac{r_f L_{it} - y_{it}f_C C_{it} - x_{it}C_{it}f_A - (1 - x_{it} - y_{it})L_{it}f_A}{2y_{it}f_A} \quad (13)$$

2.3. The determinants of the liquidity ratio

The resolution of the system formed by the three equations(11), (12) and (13) leads to the optimal level of credit supply C_{it}^* , the optimal liquidity holding L_{it}^* and the optimal demand for security assets A_{it}^* .

We determine the liquidity ratio defined here as the ratio of the optimal liquidity to the optimal credit. From the equation (12), we obtain:

$$l_{it}^* = \frac{L_{it}^*}{C_{it}^*} = \frac{2(f_C + f_A A_{it}^*/C_{it}^*)(x_{it} + y_{it} A_{it}^*/C_{it}^*)}{r_f + x_{it}(\rho + \varepsilon_{it})} \quad (14)$$

Knowing that $A_{it}^*/C_{it}^* = y_{it}/x_{it}$, it follows:

$$l_{it}^* = \frac{2\left(f_C + f_A \frac{y_{it}}{x_{it}}\right)\left(x_{it} + \frac{y_{it}^2}{x_{it}}\right)}{r_f + x_{it}(\rho + \varepsilon_{it})} \quad (15)$$

In this expression, the term y_{it}/x_{it} contains the monetary variables i_C, i_{MM}, r, p, i_D through A_{it}^*/C_{it}^* , so that the following proposition holds:

Proposition 1: The liquidity ratio depends on financial parameters, namely $x_{it}, y_{it}, f_C, f_A, r_f, \rho$. It depends also on the monetary variables i_C, i_{MM}, r, p, i_D . Formally:

$$l_{it}^* = l(i_C, i_{MM}, r, p, i_D, x_{it}, y_{it}, f_C, f_A, r_f, \rho) \quad (16)$$

Now let's introduce the liquidity ratio threshold \bar{l} . This is the liquidity ratio defined normatively as the threshold beyond which there is bank excess liquidity. Formally, there is bank excess liquidity, that is, $l_{it}^* > \bar{l}$, when:

$$\frac{\left(f_C + f_A \frac{y_{it}}{x_{it}}\right)\left(x_{it} + \frac{y_{it}^2}{x_{it}}\right)}{\bar{l}} > \frac{R_{it}}{2} \quad (17)$$

where $R_{it} = r_f + x_{it}(\rho + \varepsilon_{it})$, the return of the banking portfolio of loans and securities. $\left(f_C + f_A \frac{y_{it}}{x_{it}}\right)$ can be interpreted as the cost of converting into liquidity a portfolio consisting of a credit unit and the corresponding number of security assets. $\left(x_{it} + \frac{y_{it}^2}{x_{it}}\right)$ can be interpreted as the number of securities held in the portfolio (proportion of credits and corresponding proportion of security securities). The product of these two terms is then nothing other than the total cost of converting the bank's portfolio into liquidity. $\frac{\left(f_C + f_A \frac{y_{it}}{x_{it}}\right)\left(x_{it} + \frac{y_{it}^2}{x_{it}}\right)}{\bar{l}}$ is the average conversion cost and $\frac{R_{it}}{2}$ is the average return over the period.

When this cost is too high, which is higher than the average yield of the banking portfolio, the bank prefers to keep a lot of liquidity. Otherwise, when the average yield of the banking portfolio over the period is higher than the average cost of conversion, the bank is encouraged to invest its funds to grant more loans and acquire more securities. This results in a decrease in liquidity:

$$l_{it}^* < \bar{l} \quad \text{si} \quad \frac{\left(f_C + f_A \frac{y_{it}}{x_{it}}\right)\left(x_{it} + \frac{y_{it}^2}{x_{it}}\right)}{\bar{l}} < \frac{R_{it}}{2} \quad (18)$$

From these results follows the proposition 2:

Proposition 2: The bank has excess liquidity ($l_{it}^* > \bar{l}$) when the financial market is underperforming with respect to investments in securities and/or loans:

$$\frac{(f_C + f_A \frac{y_{it}}{x_{it}})(x_{it} + \frac{y_{it}^2}{x_{it}})}{\bar{l}} > \frac{R_{it}}{2}$$

This excess liquidity is absorbed in two cases:

- R_{it} increases, that is, when the risk-free rate r_f and/or the market risk premium ρ (here, the yield on loans) increases, ceteris paribus; cf. equation (3): $\forall i, \forall t, R_{it} = r_f + x_{it}(\rho + \varepsilon_{it})$;
- The fixed costs related to the conversion of safe securities into liquidity, f_A , and the fixed costs related to the conversion of debt securities into liquidity, f_C , fall, ceteris paribus.

In the model of the risk-neutral bank developed above, bank excess liquidity results from the optimal behavior of the bank which optimizes its profits. Excess liquidity is a sign that the financial market is not providing the right incentives for investment in securities or loan financing. Whatever the degree of economic uncertainty (low uncertainty or high uncertainty), there is excess liquidity when the total cost of conversion in liquidity of securities per unit of credit granted is greater than the expected return of the bank portfolio. So the ultimate determinant of the optimal excess liquidity here is not the uncertainty but the low expected return of the bank portfolio. The level of economic uncertainty explains indirectly the bank's optimal excess liquidity, through the idiosyncratic shock ε_{it} contained in the bank's portfolio return $R_{it} = r_f + x_{it}(\rho + \varepsilon_{it})$.

The main role played by the economic uncertainty appears in a context of high expected return of the bank portfolio. In this case, the excess liquidity is absorbed by the economy. More precisely, the bank reallocates the excess liquidity by reassigning the proportion of credit and safe securities in its portfolio. Either the bank increases the proportion of loans deemed to be riskier or it increases the proportion of safe securities. Such a reassignment requires lifting the hypothesis of a risk-neutral bank. Hence, it is assumed that the bank is risk averse, and we show how the uncertainty affects the proportions of loans and safe securities in the bank's portfolio.

3. The optimal behavior of the risk averse bank

3.1. The assumptions

The objective-function of the risk averse bank is the expected utility of its profit $U(\Pi_{it})$. As the degree of uncertainty in the economy grows, it becomes increasingly difficult to determine the optimal rate of return on loans. This pushes the bank to use an informative signal to try to predict this optimal rate of return on loans. In other words, in times of great economic uncertainty (experience of crisis, restructuring of the banking system, instability of deposits, informational asymmetry, symmetrical ignorance, weak legal, judicial and regulatory framework, etc.), signals coming from the market are ambiguous. Hence the banks use the expectation of loan return conditional on the perceived signal, to predict this optimal return. We know that the return of the portfolio of bank i in period t depends largely on ε_{it} which is realized only at the end of the period. However, it is at the beginning of the period t that each bank determines the composition of its portfolio. It does so according to the imperfect information available to it. At period t , the bank i observes an imperfect signal S_{it} allowing it to predict the value that will take the variable ε_{it} at the end of the period. This observed signal, different for each bank, is composed of a heterogeneous noise ε_{it} and a homogeneous noise v_t whose intensity $\sigma_{v_t}^2$ varies from one period to another. The homogeneous noise v_t , unlike ε_{it} , is an aggregate shock. It is assumed to be uncorrelated with ε_{it} . Formally, we have:

$$S_{it} = \varepsilon_{it} + v_t \text{ where } v_t \sim N(0, \sigma_{v_t}^2) \perp \varepsilon_{it} \quad (19)$$

In the absence of perfect information, if v_t increases, the bank observes that S_{it} increases, which causes an increase of the uncertainty on ε_{it} and therefore on the yield of the portfolio R_{it} . In other words, when the degree of uncertainty in the economy increases, the noise in the signal also increases and it becomes more and more difficult to determine the true value of ε_{it} as well as the optimal rate of return of the loans. But the bank has no other choice: to forecast ε_{it} , it needs information about ε_{it} . The best prediction of ε_{it} is its expected unconditional value (ε_{it}), which is equal to 0. But observation of the signal S_{it} can allow the bank to improve this prediction by using the expected value of ε_{it} conditional on the received signal (ε_{it}/S_{it}).

The informative nature of this signal implies that $(\varepsilon_{it}/S_{it}) \neq 0$. Suppose, like Baum and al (2002), that this conditional expectation is a proportion λ_t of the signal, namely:

$$E(\varepsilon_{it}/S_{it}) = \lambda_t S_{it} = \lambda_t (\varepsilon_{it} + v_t) \text{ where } \lambda_t = \frac{\sigma_{\varepsilon_t}}{\sigma_{\varepsilon_t}^2 + \sigma_{v_t}^2} \quad (20)$$

3.2. The optimization program of the risk averse bank

With this justification of the choice of conditional expectation in the bank's program, it follows that the objective-function of the bank is the expected utility of the conditional profit to the perceived informative signal, as in Calmès and Salazar (2006). It will be noted $E(\Pi_{it}/S_{it})$. Some restrictions on the utility function or on the prior distribution of random yields make it possible to write the objective-function above as a mean-variance function (Tobin, 1958; Markowitz, 1959; Levy-Markowitz, 1979). Thus, by noting φ , the degree of bank aversion to risk, we can write the expected utility of the conditional profit of bank i at period t , as follows:

$$EU(\Pi_{it}/S_{it}) = E(R_{it}/S_{it}) - \frac{1}{2} \varphi \text{Var}(R_{it}/S_{it}) \quad (21)$$

The expected return of the portfolio of the bank i at period t conditional on the received signal is thus written:

$$\begin{aligned} \forall i, \forall t, \quad E(R_{it}/S_{it}) &= E[r_f + x_{it}(\rho + \varepsilon_{it})/S_{it}] \\ \forall i, \forall t, \quad E(R_{it}/S_{it}) &= r_f + x_{it}[\rho + E(\varepsilon_{it}/S_{it})] = r_f + x_{it}(\rho + \lambda_t S_{it}) \end{aligned} \quad (22)$$

It is shown that the conditional variance of the return of this banking portfolio is (proof in appendix A1):

$$\forall i, \forall t, \quad \text{Var}(R_{it}/S_{it}) = \lambda_t \sigma_{\varepsilon_t}^2 x_{it}^2 \quad (23)$$

The objective function of the bank is then written:

$$EU(\Pi_{it}/S_{it}) = r_f + x_{it}(\rho + \lambda_t S_{it}) - \frac{1}{2} \varphi \lambda_t \sigma_{\varepsilon_t}^2 x_{it}^2 \quad (24)$$

Maximizing with respect to x_{it} , the first order conditions are (proof in appendix A2):

$$\forall i, \forall t, \quad x_{it} = \frac{\rho + \lambda_t S_{it}}{\varphi \lambda_t \sigma_{\varepsilon_t}^2} \quad (25)$$

$$\forall i, \forall t, \quad \text{Var}(x_{it}) = \frac{\sigma_{\varepsilon_t}^2 + \sigma_{v_t}^2}{\varphi^2 \sigma_{v_t}^4} \quad (26)$$

3.3. Absorption of excess liquidity in a context of uncertainty

Let us now show that economic uncertainty is also reflected in a decrease in the supply of funds of the bank. For this, we determine the direction of the relationship between the homogeneous aggregate shock and the proportion of loans in the bank's portfolio:

$$\frac{\partial x_{it}}{\partial \sigma_{v_t}^2} = -\frac{(\rho + \lambda_t S_{it})}{\varphi \lambda_t \sigma_{v_t}^4} < 0 \quad (27)$$

When the degree of uncertainty in the economy $\sigma_{v_t}^2$ increases, the proportion x_{it} of funds invested in loans decreases in favor of safe securities. Indeed, with the increase of the noise in the signal, it becomes more and more difficult to determine the true value of ε_{it} as well as the optimal rate of return of the loans. In this case, when the risk perceived by the bank exceeds a signal that it considers acceptable, it is encouraged to lower the proportion of funds invested in loans and to increase that invested in safe securities. It is also possible to evaluate the impact of uncertainty on the variance of the ratio of loans to banks' assets:

$$\frac{\partial \text{Var}(x_{it})}{\partial \sigma_{v_t}^2} = -\frac{1}{\varphi^2} \left[\frac{2\sigma_{\varepsilon_t}^2}{\sigma_{v_t}^6} + \frac{1}{\sigma_{v_t}^4} \right] < 0 \quad (28)$$

As the level of uncertainty in the economy increases, so does the variance in the ratio of loans to total assets. In this case, the bank reallocates the excess liquidity by reassigning the proportion of credit and safe securities in its portfolio. More precisely, the bank decreases the proportion of loans deemed to be riskier and increases the proportion of safe securities.

These theoretical results are consistent with the results of various empirical studies (Sigouin, 2003; Calmès, 2004; Beguy, 2012). The excess liquidity does not systematically go to the financing of the economy. It can be mainly invested in safe securities. We summarize these results in the following proposition.

Proposition 3: In a context of high expected return of the bank portfolio, when the degree of uncertainty in the economy $\sigma_{v_t}^2$ increases, the proportion x_{it} of funds invested in loans decreases in favor of securities security. As the level of uncertainty in the economy increases, so does the variance in the ratio of loans to total assets. This means that banks tend to choose portfolios that are similar in terms of asset allocation. This leads to a decrease in the overall supply of funds on the market.

4. Concluding remarks

In this paper, we have developed a theoretical model in which excess banking liquidity results from an optimal behavior of a risk neutral profit-maximizing bank. Whatever the degree of economic uncertainty (low uncertainty or high uncertainty), there is excess liquidity if the expected return of the banking portfolio (safe securities, loan financing, etc.) is too low. In order to overcome the excess liquidity in countries facing a subfinancing of the economy, the regulator can implement incentive measures that reinforce the return of bank portfolios. Put another way, an economic policy implication is that the absorption of excess liquidity does not only go through policies aimed at minimizing uncertainty but especially by measures to strengthen financial market incentives for bank portfolios. Examples of such measures are: setting a satisfactory level of credit cost for banks through monetary policy; setting a satisfactory level of risk free rate; reduced costs of bank deposits; reduced fixed costs related to the conversion of safe securities into liquidity; reduces fixed costs related to the conversion of debt securities into liquidity; etc.

In our model, the uncertainties play an active role in a context of high expected return of the bank portfolio. In this case, the excess liquidity decreases but it is not systematically oriented towards the financing of the economy. In order to drain excess liquidity towards the financing of private sector of the economy, the regulator should enforce measures that minimize the economic uncertainties. Knowing that uncertainty increases the credit risk, the regulator can apply measures to control credit risk so that the excess liquidity is oriented more on financing the economy than on investing in safe securities. Such measures concern the development of insurance products (credit insurance, credit derivatives, etc.), the establishment of specific guarantee funds that can absorb the banks' excess liquidity by allowing them to recover a portion of their receivables in the event of default. Another measure is the tax bonus which encourages, via tax give aways, banks that are more involved in the financing of the economy. An extension of this work could be to empirically test the hypothesis that, for banks considered to hold excess liquidity, one would expect to find that the expected return on their banking portfolio is low. And for the others, the expected return on their banking portfolio is sufficiently high. Further study is expected in the future.

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Appendix A1

Let's prove that $\forall i, \forall t, \text{Var}(R_{it}/S_{it}) = \lambda_t \sigma_{\varepsilon_t}^2 x_{it}^2$.

The return of the banking portfolio is $R_{it} = r_f + x_{it}(\rho + \varepsilon_{it})$ so that the conditional variance of ε_{it} is :

$$\begin{aligned} & \left[\right. \\ & \forall i, \forall t, \text{Var}(R_{it}/S_{it}) = \text{Var} r_f + x_{it}(\rho + \varepsilon_{it})/S_{it} \\ & \quad = \text{Var}(r_f/S_{it}) + x_{it}^2 \text{Var}(\rho/S_{it}) + x_{it}^2 \text{Var}(\varepsilon_{it}/S_{it}) \quad (A1.1) \end{aligned}$$

We have $\text{Var}(r_f/S_{it}) = 0$ since r_f is certain and $\text{Var}(\rho/S_{it}) = 0$ since ρ is constant, so that :

$$\forall i, \forall t, \text{Var}(R_{it}/S_{it}) = x_{it}^2 \text{Var}(\varepsilon_{it}/S_{it}) \quad (A1.2)$$

As the conditional expectation of ε_{it} is a proportion λ_t of the signal S_{it} , namely $(\varepsilon_{it}/S_{it}) = \lambda_t S_{it}$, the conditional variance of ε_{it} is a proportion λ_t of its unconditional variance:

$$\text{Var}(\varepsilon_{it}/S_{it}) = \lambda_t \text{Var}(\varepsilon_{it}) = \lambda_t \sigma_{\varepsilon_t}^2 \quad (A1.3)$$

Finally:

$$\forall i, \forall t, \text{Var}(R_{it}/S_{it}) = x_{it}^2 \text{Var}(\varepsilon_{it}/S_{it}) = x_{it}^2 \lambda_t \sigma_{\varepsilon_t}^2 \quad QED \quad (A1.4)$$

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Appendix A2

Maximizing the objective function of the bank $EU(\Pi_{it}/S_{it}) = r_f + x_{it}(\rho + \lambda_t S_{it}) - \frac{1}{2}\varphi\lambda_t\sigma_{\varepsilon_t}^2 x_{it}^2$ with respect to x_{it} , the first order condition gives:

$$\begin{aligned}\frac{dEU(\Pi_{it}/S_{it})}{dx_{it}} &= \rho + \lambda_t S_{it} - \varphi\lambda_t\sigma_{\varepsilon_t}^2 x_{it} = 0 \\ \Rightarrow x_{it} &= \frac{\rho + \lambda_t S_{it}}{\varphi\lambda_t\sigma_{\varepsilon_t}^2} \quad (A2.1)\end{aligned}$$

From equation A2.1, we obtain:

$$Var(x_{it}) = Var\left(\frac{\rho + \lambda_t S_{it}}{\varphi\lambda_t\sigma_{\varepsilon_t}^2}\right) = \frac{1}{\varphi^2\lambda_t^2\sigma_{\varepsilon_t}^4} Var(\rho + \lambda_t S_{it}) \quad (A2.2)$$

We have $Var(\rho + \lambda_t S_{it}) = \lambda_t^2 Var(S_{it}) = \lambda_t^2 Var(\varepsilon_{it} + v_t) = \lambda_t^2 [Var(\varepsilon_{it}) + Var(v_t)]$ since $\varepsilon_{it} \perp v_t$.
Thus:

$$Var(\rho + \lambda_t S_{it}) = \lambda_t^2 (\sigma_{\varepsilon_t}^2 + \sigma_{v_t}^2) \quad (A2.3)$$

Finally :

$$Var(x_{it}) = \frac{\lambda_t^2}{\varphi^2\lambda_t^2\sigma_{\varepsilon_t}^4} (\sigma_{\varepsilon_t}^2 + \sigma_{v_t}^2) = \frac{\sigma_{\varepsilon_t}^2 + \sigma_{v_t}^2}{\varphi^2\sigma_{\varepsilon_t}^4} \quad QED \quad (A2.4)$$